QUESTION 1: Government deficits (12 points):

Consider an economy described as follows:
$C = C_a + c(Y-T) = 750 + 0.8(Y-T)$
$T = T_a + t*Y = 100 + 0.1Y$,
$G = 300$,
$I_P = 400$,
$NX = NX_a - nx*Y = 230 – 0.12*Y$.

(A) Assume that the economy described above is at natural GDP. What is the value of the structural government deficit/surplus? (4 points)

$A_p = 750 – 0.8 * (100) + 400 + 300 + 230 = 1600$
$k = 1/((1-0.8)*(1-0.1) + 0.1 + 0.12) = 2.5$
$Y = k* A_p = 2.5 * 1600 = 4000$
$T = 100 + 0.1*4000 = 500$
$T-G = 500 –300 = 200$

(B) Suppose that I_p temporarily falls to 100 and we are no longer at natural GDP. What is the value of the actual deficit/surplus? (2 points)

$A_p = 750 – 0.8 * (100) + 100 + 300 + 230 = 1300$
$Y = k* A_p = 2.5 * 1400 = 3250$
$T = 100 + 0.1*3250 = 425$
$T-G = 425 – 300 = 125$

(C) The government wants to bring actual deficit/surplus back to the level of part A. What is the level of government expenditures needed to achieve this goal? *Hint: this question is harder than it looks, because when G changes, also Y and T change. You should use a system of three equations in three unknowns (Y, T and G) and find the value of G that solves it. The first equation is the tax function; the second is the equation for the equilibrium level of output, where you should leave G unknown; the last is an equation for the actual deficit target where you set T-G equal to the value you calculated in part A.* (6 points)

$T = 100 + 0.1Y$
$Y = 2.5 * (1000+G) = 2500 + 2.5*G$
$T – G = 200$

The solution is G = 200.

QUESTION 2: Growth Rates (8 points)

(a) Suppose real GDP in Country A was 200 in 2013 and 220 in 2015. If real GDP grew from 2015 to 2017 at the same growth rate from 2013 to 2015, what was the real GDP in Country A in 2017? (4 Points)

Solution:
First, solve for growth rate between 2013-2015:

$$g_{(2013-2015)} = \frac{ln\frac{GDP_{2015}}{GDP_{2013}}}{2015-2013} = \frac{ln\frac{220}{200}}{2} = 4.8\%$$

Next, solve for real GDP in 2017:

$$GDP_{2017} = GDP_{2015}\left(e^{0.048(2017-2015)}\right) = 220(1.1) = 242$$

(b) Suppose Country B has a real GDP of 150 in 2013. The government in Country B wants to grow their real GDP at a continuous rate until they reach the same real GDP as Country C in 2021. Assuming that real GDP in Country C in 2017 is 230 and it grows at an annualized continuous rate of 3% after 2017, what annualized rate should the government in Country B target? (4 Points)

Solution:
First, solve for GDP in country C in 2021:

$$GDP_{2021,C} = GDP_{2017,C}\left(e^{0.03(2021-2017)}\right) = 230(1.13) = 260$$

Next, solve for the annualized growth rate of GDP in county B assuming that $GDP_{2021,B} = GDP_{2021,C}$:

$$g = \frac{ln\frac{GDP_{2021,C}}{GDP_{2013,B}}}{2021-2013} = \frac{ln\frac{260}{150}}{8} = 6.8\%$$

QUESTION 3: SP-DG Model (10 points):

Suppose that the following equations describe an economy currently at long-run equilibrium:

$$p_t = p_t{}^e + 0.5 \cdot \hat{Y}_t + z_t$$
$$p_t{}^e = 0.5 \cdot p_{t-1}{}^e + 0.5 \cdot p_{t-1}$$
$$\hat{Y}_0 = 0, \hat{x}_0 = 3, p_0{}^e = 3, p_0 = 3, z_0 = 0.$$

(a) Write down the SP and DG equations using the information above. (2 point)

SP	**Solution:** For the SP just plug the equation for $p_t{}^e$ into the first equation: $$p_t = 0.5 \cdot p_{t-1}{}^e + 0.5 \cdot p_{t-1} + 0.5 \cdot \hat{Y}_t + z_t$$
DG	**Solution:** The DG equation is always the same: $$\hat{Y}_t = \hat{Y}_{t-1} + \hat{x}_t - p_t$$

(b) Substitute the DG equation into the numerical SP equation and solve for p_t as a function of $p_{t-1}, p_{t-1}^e, \hat{Y}_{t-1}, \hat{x}_t$, and z_t. (3 point)

Solution:
Follow the instructions:

$$p_t = 0.5 \cdot p_{t-1}{}^e + 0.5 \cdot p_{t-1} + 0.5 \cdot \hat{Y}_t + z_t \Rightarrow$$
$$p_t = 0.5 \cdot p_{t-1}{}^e + 0.5 \cdot p_{t-1} + 0.5 \cdot \left[\hat{Y}_{t-1} + \hat{x}_t - p_t\right] + z_t \Rightarrow$$
$$1.5p_t = 0.5 \cdot p_{t-1}{}^e + 0.5 \cdot p_{t-1} + 0.5 \cdot \left[\hat{Y}_{t-1} + \hat{x}_t\right] + z_t \Rightarrow$$
$$p_t = \frac{1}{3} \cdot p_{t-1}{}^e + \frac{1}{3} \cdot p_{t-1} + \frac{1}{3} \cdot \hat{Y}_{t-1} + \frac{1}{3} \cdot \hat{x}_t + \frac{2}{3} \cdot z_t$$

Give full credit even if students don’t multiply through all the fractions

NOTE: The following question asks you to compute the path of the system given an initial shock. In order to earn partial credit for these parts you must show your work in the space provided. By “show your work" we mean that the grader should be able to understand a) what you are computing and b) where the numbers in your computation are coming from.

(c) Starting in the long-run equilibrium described above in period 0, assume that in period t=1 we observe a temporary shock to z_t. In particular, $z_1 = 2, z_2 = 0$. Fill in the following table assuming that the central bank is following an accommodating policy. (5 points)

t	${p_t}^e$	$\hat{Y}_t$	$\hat{x}_t$	p_t	z_t
0	3	0	3	3	0
1	**3**	**0**	**2.5**	**2.5**	2
2	**2.75**	**0**	**1.375**	**1.375**	0

Note: since the central bank is following an accommodative policy, we know $\hat{Y}_t = 0$ in each period. Since from the DG equation $\hat{Y}_t = \hat{Y}_{t-1} + \hat{x}_t - p_t$, we know that $\hat{x}_t = p_t$ in each period. We thus have

$$p_t = \frac{1}{3} \cdot {p_{t-1}}^e + \frac{1}{3} \cdot p_{t-1} + \frac{1}{3} \cdot \hat{Y}_{t-1} + \frac{1}{3} \cdot p_t + \frac{2}{3} \cdot z_t$$
$$\frac{4}{3} p_t = \frac{1}{3} \cdot {p_{t-1}}^e + \frac{1}{3} \cdot p_{t-1} + \frac{1}{3} \cdot \hat{Y}_{t-1} + \frac{2}{3} \cdot z_t$$
$$p_t = \frac{1}{4} \cdot {p_{t-1}}^e + \frac{1}{4} \cdot p_{t-1} + \frac{1}{4} \cdot \hat{Y}_{t-1} + \frac{1}{2} z_t$$

Since $\hat{Y}_t = 0$ in each period, we have

$$p_t = \frac{1}{4} \cdot {p_{t-1}}^e + \frac{1}{4} \cdot p_{t-1} + \frac{1}{2} z_t$$

QUESTION 4: Open IS-LM model (15 points):

Let the following represent the structure of a **small open economy** with **perfect capital mobility.** Suppose the economy starts with a **flexible exchange rate** regime.

$C = 200 + 0.7(Y\text{-}T)$,
$T = 50+0.1Y$,
$G=70$,
$I_P = 70 - 10r$,
$NX = 115 - 0.13Y - 10e$,
$(M/P)^D = 0.2Y - 5r$,
$M^S/P = 100$.

(a) Assume that initially foreign and domestic interest rates are equal so that $r = r^f$ and let the foreign exchange rate e=2. Find the IS and LM equations. (3 points)

$k=1/((1-0.7)\times(1-0.1) + 0.1 + 0.13)=1/0.4 = 2$

$Ap=200 - 0.7*50 + 70 +70 - 10r + 115 - 10e = 420 - 10r - 10e = 400\text{-}10r$

IS: $Y = 2*(400\text{-}10r)=800 - 20r$
LM: $100 = 0.25Y - 5r \to Y = 400 + 20r$
(b) Find the equilibrium income, interest rate and net exports. (2 points)

$400 + 20r = 800 - 20r \to 40r = 400 \to r = 10 \to Y = 600$
$NX = 115 - 0.13*600 - 20 = 17$

(c) Suppose autonomous consumption suddenly goes up from 200 to 250.

(c1) Write down the new IS curve, after the shift in autonomous planned consumption. Keep in mind that this is a small open economy with **perfect capital mobility** and **flexible exchange rates**. *Hint: Express both Ap and Y in terms of r and e; don't solve for e* (4 points)

$Ap=250 - 0.7*50 + 70 +70 - 10r + 115 - 10e = 470 - 10r - 10e$
New "IS": $Y=k*Ap=2(370 - 10r - 10e) = 940 - 20e - 20r$

(c2) Use the new IS curve and the LM curve to calculate the new output and exchange rate. *Hint: because this is an open economy with perfect capital mobility, the interest rate does not change from what you calculated in part (b). Use the LM curve to calculate real GDP and from this calculate the new value of the exchange rate.* (6 points)

LM: $Y = 400 + 20r \to r=10, Y=600$
"IS": $Y = 940 - 20e - 20r = 740 - 20e = 600$
$20e=140 \to e=7$

QUESTION 5: Solow model (15 points)

Suppose the production function of a closed economy is given by $Y = AN^{2/3}K^{1/3}$, where K is capital, N is labor and A=20 is total factor productivity.

Furthermore, suppose that capital depreciates at d=0.1, the saving rate is s=0.2, and population growth is n=0.15.

Assume that T=G=NX=Ca=0.

(A) Find the steady state level of capital per capita. (3 points)

$Y/N = A(K/N)^{1/3}$
$s^* A(K/N)^{1/3} = (n+d)^*(K/N)$ -> $K/N = (sA/(n+d))^{3/2} = (16)^{3/2} = 64$

(B) Find the steady state level of output per capita and consumption per capita. (2 points)

$Y/N = A(K/N)^{1/3} = 20 * 4 = 80$
$C/N = (1-s)*Y/N = 0.8 * 80 = 64$

(C) Suppose the **depreciation rate (d) decreases**. Would you expect steady state consumption per capita to go up or down or is the answer ambiguous and depends on the exact change in *n*? Show the effect on the graph and briefly explain your answer. (4 points)

Up

(D) **When you solve this part, ignore part C**. Now assume saving rate goes up to **s=0.25**. Solve for the steady-state consumption per capita (C/N=?). Does it go up or down comparing to part B?(3 points)

$s^* A(K/N)^{1/3} = (n+d)^*(K/N)$ -> $K/N = (sA/(n+d))^{3/2} = (20)^{3/2} = 89.4$

$C/N = (1-s)\, A(K/N)^{1/3} = 0.75 * 20 * 89.4^{1/3} = 67.07$

C/N goes up.

(E) **When you solve this part, ignore part C**. Now assume saving rate goes up to **s=0.5**. Solve for the steady-state consumption per capita (C/N=?). Does it go up or down comparing to part B? (3 points)

$s^* A(K/N)^{1/3} = (n+d)^*(K/N)$ -> $K/N = (sA/(n+d))^{3/2} = (40)^{3/2} = 253$

$C/N = (1-s)\, A(K/N)^{1/3} = 0.5 * 20 * 148.2^{1/3} = 63.24$

C/N (slightly) goes down (We can accept if they say stays same).